Discrete Mathematics 348 (2025) 114383

Contents lists available at ScienceDirect

Discrete Mathematics

journal homepage: www.elsevier.com/locate/disc

Note

The sandwich problem for odd-hole-free and even-hole-free graphs

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ARTICLE INFO

Article history:
Received 16 April 2024
Received in revised form 10 December 2024
Accepted 20 December 2024
Available online xxxx

Keywords:
05C85
Graph sandwich problems
Forbidden induced subgraphs
Even-hole-free graphs
Odd-hole-free graphs

ABSTRACT

For a property $\mathcal{P}$ of graphs, the $\mathcal{P}$-Sandwich-Problem, introduced by Golumbic and Shamir (1993), is the following: Given a pair of graphs (G_1, G_2) on the same vertex set V, does there exist a graph G such that $V(G) = V$, $E(G_1) \subseteq E(G) \subseteq E(G_2)$, and G satisfies $\mathcal{P}$? A *hole* in a graph is an induced subgraph which is a cycle of length at least four. An odd (respectively even) hole is a hole of odd (respectively even) length. Given a class of graphs $\mathcal{C}$ and a graph G we say that G is *$\mathcal{C}$-free* if it contains no induced subgraph isomorphic to a member of $\mathcal{C}$. In this paper we prove that if $\mathcal{P}$ is the property of being odd-hole-free or the property of being even-hole-free, then the $\mathcal{P}$-Sandwich-Problem is NP-complete.

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1. Introduction

All graphs in this paper are finite and simple. Let G be a graph. G^c denotes the *complement* of G, obtained from G by replacing each edge with a non-edge and vice versa.

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$. Then G_2 is a *supergraph* of G_1 if $V_1 = V_2 = V$ and $E_1 \subseteq E_2$. A pair (G_1, G_2) of graphs so that G_2 is a supergraph of G_1 is called a *sandwich instance*. The edges in E_1 are called *forced*, while the edges in $E_2 \setminus E_1$ are *optional*. We let E_3 be the set of all edges in the complete graph with vertex set V which are not in E_2, and call them *forbidden* edges. A graph G is called a *sandwich graph* for the sandwich instance (G_1, G_2) if G_2 is a supergraph of G and G is a supergraph of G_1.

For a property $\mathcal{P}$ of graphs, the $\mathcal{P}$-Recognition-Problem is the problem of deciding whether a given graph G satisfies $\mathcal{P}$. The $\mathcal{P}$-Sandwich-Problem is the following: For a given sandwich instance (G_1, G_2), does there exist a sandwich graph

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[1] We acknowledge the support of the Natural Sciences and Engineering Research Council of Canada (NSERC), [funding reference number RGPIN-2016-06517]. Cette recherche a été financée par le Conseil de Recherches en Sciences Naturelles et en Génie du Canada (CRSNG), [numéro de référence RGPIN-2016-06517].

[2] Supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico of Brazil (CNPq) grant 305356/2021-6.

[3] We acknowledge the support of the Natural Sciences and Engineering Research Council of Canada (NSERC), [funding reference number RGPIN-2020-03912]. Cette recherche a été financée par le Conseil de Recherches en Sciences Naturelles et en Génie du Canada (CRSNG), [numéro de référence RGPIN-2020-03912]. This project was funded in part by the Government of Ontario. This research was conducted while Spirkl was an Alfred P. Sloan Fellow.

https://doi.org/10.1016/j.disc.2024.114383
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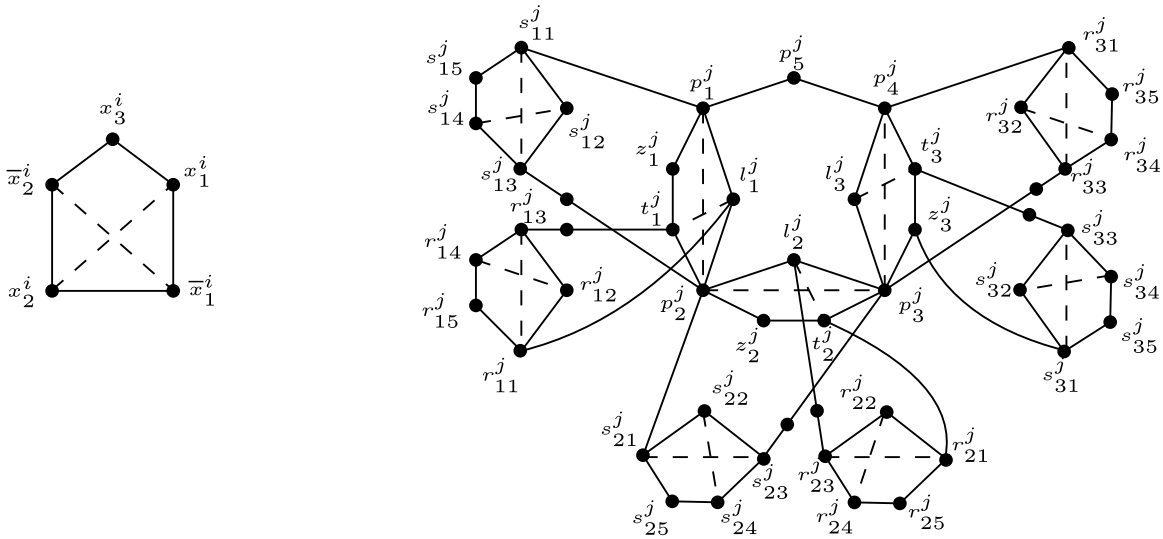

Fig. 1. This figure illustrates the gadgets of the construction that we discuss in the proof of Theorem 2.1. The variable gadget for a variable x_i is shown on the left; and the clause gadget for a clause c_j is shown on the right. Edges represented with dashed line segments are optional edges, that is, edges of $E_2 \setminus E_1$; the solid edges are forced, that is, edges of E_1. The omitted edges are forbidden, that is, edges of E_3.

G for (G_1, G_2) so that G satisfies $\mathcal{P}$? This generalization of the recognition problem was introduced by Golumbic and Shamir [10]. If the $\mathcal{P}$-Recognition-Problem is in P or NP, then the $\mathcal{P}$-Sandwich-Problem is clearly in NP. The sandwich problem becomes the recognition problem when $G_1 = G_2$, and thus, if the $\mathcal{P}$-Recognition-Problem is NP-complete, so is the $\mathcal{P}$-Sandwich-Problem.

Let $\mathcal{P}$ be a graph property. We define the complementary property $\mathcal{P}^c$ by saying that G satisfies $\mathcal{P}^c$ if and only if G^c satisfies $\mathcal{P}$. Note that the $\mathcal{P}$-Sandwich-Problem is invariant under taking complements in the following sense.

Lemma 1.1. *The $\mathcal{P}^c$-Sandwich-Problem is NP-hard if and only if the $\mathcal{P}$-Sandwich-Problem is NP-hard.*

Proof of Lemma 1.1. An instance (G_1, G_2) is a Yes instance for $\mathcal{P}^c$-Sandwich-Problem if and only if (G_2^c, G_1^c) is a Yes instance for the $\mathcal{P}$-Sandwich-Problem. □

We let P_k denote an induced path on k vertices, and C_k denote an induced cycle on k vertices. An induced cycle C_k with $k \geq 4$ vertices is called a *hole*; it is called an *odd hole* if k is odd, and an *even hole* if k is even. An *antihole* is the complement of a hole. It is an *odd antihole* if its complement is an odd hole, and an *even antihole* otherwise. Let $\mathcal{C}$ be a set of graphs. We say that G is *$\mathcal{C}$-free* if no induced subgraph of G is isomorphic to a graph in $\mathcal{C}$.

A graph is *Berge* if it contains no odd hole and no odd antihole as an induced subgraph. A graph G is *perfect* if for each induced subgraph H of G, the clique number of H equals the chromatic number of H. The Strong Perfect Graph Theorem [5], first conjectured in [1], states that a graph is perfect if and only if it is Berge. It is known that Berge graphs can be recognized in polynomial time [3]. Research has focused on the $\mathcal{P}$-Sandwich-Problem for subclasses $\mathcal{P}$ of perfect graphs, and for decompositions related to perfect graphs. The complexity of the Perfect-Graph-Sandwich-Problem remains one of the most prominent open questions in this area [9].

This paper is organized as follows. In Section 2, we show that the hardness reduction that proves that the C_5-Free-Sandwich-Problem is NP-hard [8] actually proves that the Odd-Hole-Free-Sandwich-Problem is NP-hard, and thus NP-complete. In Section 3, we modify the hardness reduction that proves that the Chordal-Sandwich-Problem is NP-hard [2] to prove that the Even-Hole-Free-Sandwich-Problem is NP-hard, and thus NP-complete. In Section 4, we give our concluding remarks.

2. The Odd-Hole-Free-Sandwich-Problem

We show that the hardness reduction given in [8] that proves that the C_5-free-Sandwich-Problem is NP-hard actually proves that the Odd-Hole-Free-Sandwich-Problem is NP-hard, and thus NP-complete. For the convenience of the reader, in the present section, we use the same notation of [8].

Theorem 2.1. *The* Odd-Hole-Free-Sandwich-Problem *is NP-complete.*

Proof of Theorem 2.1. By [6], there is a polynomial-time algorithm for recognizing odd-hole-free graphs, and therefore, the Odd-Hole-Free-Sandwich-Problem is in NP. In what follows we prove that the Odd-Hole-Free-Sandwich-Problem is NP-hard. First we briefly describe a construction given in [8, see Theorem 2 and Figure 5] that establishes that the C_5-free-Sandwich-Problem is NP-hard (Fig. 2). Let (U, C) be a 3-sat instance with a set $U = \{x_1, \ldots, x_n\}$ of variables and a set $C = \{c_1, \ldots, c_m\}$ of clauses such that each clause contains exactly three variables.

For every variable x_i of (U, C), we will define a set $X_i = \{x_1^i, \overline{x_1}^i, x_2^i, \overline{x_2}^i, x_3^i\}$, which consists of a five-cycle of forced edges of E_1, along with two optional edges of E_2, which are chords of this five-cycle and form a matching. At least one of these optional edges is present in every C_5-free sandwich graph for our instance, and which of the chords is present will correspond to whether x_i is true or false. For every clause c_j of (U, C), and every literal $\ell_q^j \in \left\{x_i, \overline{x_i}\right\}$ in c_j, we add two gadgets $\left\{r_{q1}^j, \ldots, r_{q5}^j\right\}$ and $\left\{s_{q1}^j, \ldots, s_{q5}^j\right\}$, each a five-cycle of forced edges along with two optional edges, which are designed to provide a copy of x_i if x_i is true and if x_i is false, respectively.[4] For c_j, we add a five-cycle with vertex set $\left\{p_1^j, \ldots, p_5^j\right\}$, which has one optional edge of E_2 for each literal, and two forced edges of E_1, we also add the vertices $\cup_{q\in\{1,2,3\}}\{l_q^j, t_q^j, z_q^j\}$, and finally for each $q \in \{1, 2, 3\}$, we add forced edges so that $p_q^j, z_q^j, t_q^j, p_{q+1}^j, l_q^j$ is a five-cycle of forced edges, and we add the optional edge $t_q^j l_q^j$ in E_2 (see Fig. 1). It follows that if there is a set of optional edges that we can add to create a C_5-free sandwich graph G, then there is a truth assignment in which for every clause c_j, not all three optional edges among $\left\{p_1^j, \ldots, p_5^j\right\}$ are present in G, and hence c_j contains a true literal.

It is clear from the construction that for every 3-SAT instance in which each clause contains exactly three variables, the graph G_1 of forced edges is triangle-free (see Fig. 3). The graph of optional edges is a forest, and each connected component consists either of an edge, a single vertex, or a three-edge path. So every triangle in G_2 contains exactly one optional edge and two forced edges.

Every triangle in G_2 shares exactly one edge with another triangle in G_2, and this edge is a forced edge shared by exactly two triangles, and hence the graph G_2 does not contain as a (non-induced) subgraph the graph $\{P_4 + P_1\}^c$, because the latter contains a triangle sharing two of its edges with other triangles (see Fig. 4). Since P_6^c contains $\{P_4 + P_1\}^c$ as an induced subgraph, by the above observation, it follows that every sandwich graph G for the constructed instance (G_1, G_2) is P_6^c-free. Hence, every sandwich graph G for the constructed instance (G_1, G_2) is C_k^c-free, for $k \geq 7$. Therefore, the Odd-Antihole-Free-Sandwich-Problem is NP-complete. By Lemma 1.1, we obtain that the Odd-Hole-Free-Sandwich-Problem is NP-complete. □

3. The Even-Hole-Free-Sandwich-Problem

For the convenience of the reader, in the present section, we use the same notation of [2]. We shall modify a construction given in [2] that establishes that the Chordal-Sandwich -Problem is NP-complete in order to prove the following:

Theorem 3.1. *The* Even-Hole-Free-Sandwich-Problem *is NP-complete.*

Proof of Theorem 3.1. By [7], even-hole-free graphs can be recognized in polynomial time; it follows that the Even-Hole-Free-Sandwich-Problem is in NP. In what follows we prove that the Even-Hole-Free-Sandwich-Problem is NP-hard. First we briefly describe a construction given in [2] that establishes that the chordal graph sandwich problem is NP-hard. Let (U, C) be a 3-SAT instance with a set U of variables and a set C of clauses such that each clause contains exactly three variables. We assume that no clause contains both a variable and its complement. We have a vertex H called the head and a vertex F called the foot. For each variable X, we have two vertices S_X and $S_{\overline{X}}$, called *shoulders*. For each variable X and each clause i containing either X or $\overline{X}$, we have two vertices K_X^i and $K_{\overline{X}}^i$, called *knees*. These are the vertices in the vertex set V of the constructed instance.

The forbidden edges in E_3 are HF, $S_X S_{\overline{X}}$, $K_X^i K_{\overline{X}}^i$, for $X \in U$ and clauses i containing X or $\overline{X}$, so E_3 is a matching. For each variable X and each clause i containing either X or $\overline{X}$, we will have a six-cycle $H, S_X, K_{\overline{X}}^i, F, K_X^i, S_{\overline{X}}$ of forced edges in E_1. For each clause $i = (X + Y + Z)$, we will have a forced 3-sun with triangle $\left\{K_X^i, K_Y^i, K_Z^i\right\}$, and forced pendant edges $K_{\overline{Y}}^i K_X^i, K_{\overline{Z}}^i K_Y^i, K_{\overline{X}}^i K_Z^i$ (see Fig. 5). The vertices K_X^i, K_Y^i, K_Z^i are called *active knees*, and the vertices $K_{\overline{X}}^i, K_{\overline{Y}}^i, K_{\overline{Z}}^i$ are called *inactive knees*. All remaining edges in the complete graph with vertex set V which are not in $E_1 \cup E_3$ are optional edges. In what follows we refer to the graphs (V, E_1) and (V, E_2) as G_1 and G_2 respectively.

As shown in [2], there is a satisfying truth assignment for (U, C) if and only if there is a sandwich graph G for the sandwich instance (G_1, G_2), such that G is chordal. As noted in [2], there are exactly two ways to add optional edges to the graph G_1 in order to triangulate each forced six-cycle corresponding to a variable: the *positive* orientation adds $HK_X^i, K_X^i S_X, S_X F$ or the *negative* orientation adds $HK_{\overline{X}}^i, K_{\overline{X}}^i S_{\overline{X}}, S_{\overline{X}} F$. Further, all six-cycles corresponding to variable X must be oriented in the same way, since otherwise the set $\{H, S_X, S_{\overline{X}}, F\}$ induces a four-cycle. Therefore, given a chordal completion G we may

[4] We remark that the auxiliary vertices with no label are in a two-edge path needed to connect the gadgets.

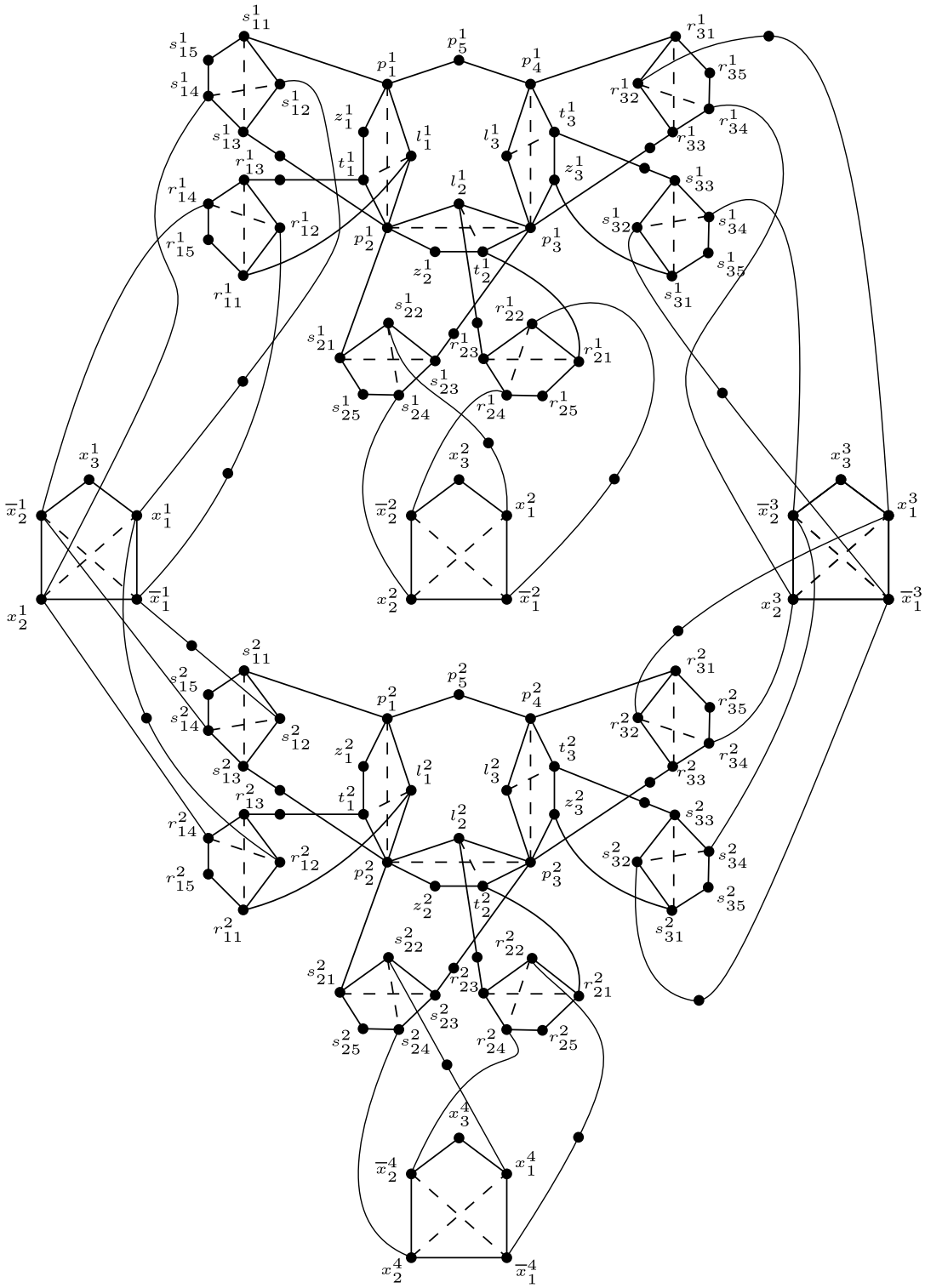

Fig. 2. This figure shows a specific instance for the C_5-FREE-SANDWICH-PROBLEM which has been created, using the construction that we discuss in the proof of Theorem 2.1, from the following instance of the 3-SAT problem: $(U, C) = \{\{x_1, x_2, x_3, x_4\}, \{(x_1 \vee x_2 \vee \overline{x}_3), (\overline{x}_1 \vee \overline{x}_3 \vee x_4)\}\}$.

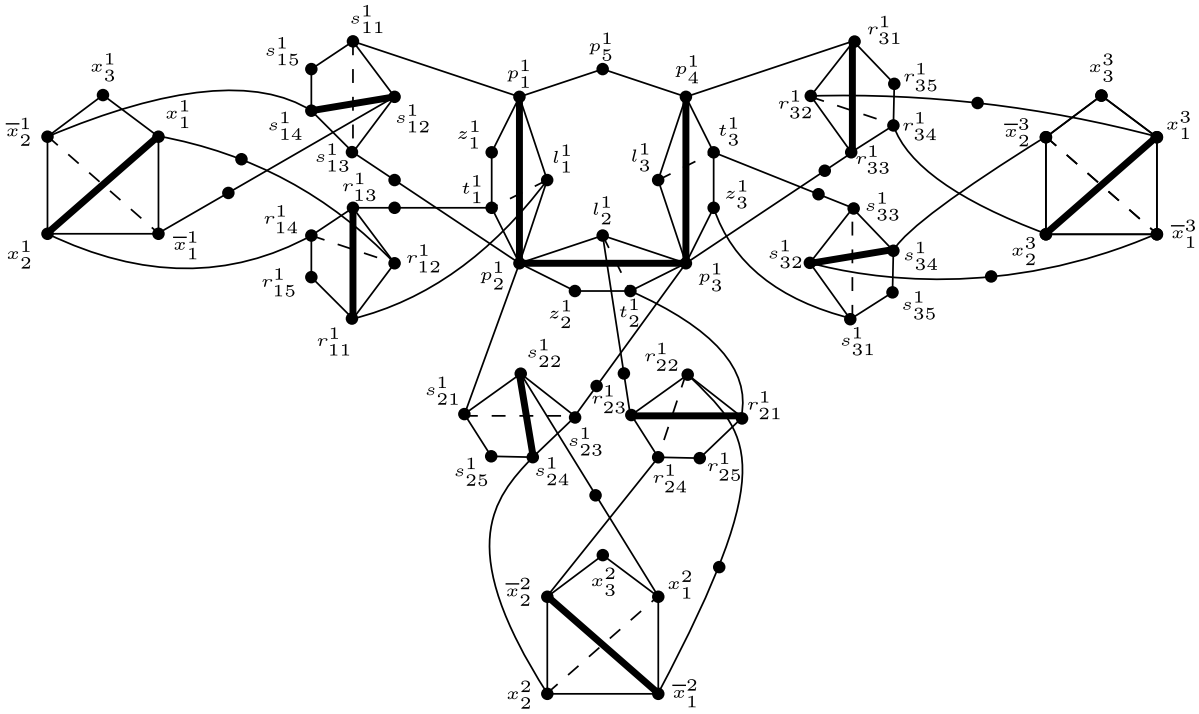

Fig. 3. This figure illustrates a sandwich graph for the instance of the C_5-FREE-SANDWICH-PROBLEM which has been created following the procedure that we discuss in the proof of Theorem 2.1, from the following instance of the 3-SAT problem: $(U, C) = \{\{x_1, x_2, x_3\}, \{(\overline{x}_1 \vee x_2 \vee \overline{x}_3)\}\}$. The thick edges are the chosen optional edges. The sandwich graph which is illustrated corresponds to the truth assignment in which the variables x_1 and x_3 are true, and the variable x_2 is false. We remark that for each false literal $l^1_j \in \{x_i, \overline{x}_i\}$ the edge $\{r^1_{j1}, r^1_{j3}\}$ is forced by the optional edge which correspond to the truth assignment of the variable x_i, and in turn the edge $\{r^1_{j1}, r^1_{j3}\}$ forces the optional edge $\{p^1_j, p^1_{j+1}\}$. Since all three literals are false in this truth assignment, all the optional edges $\{p^1_1, p^1_2\}, \{p^1_2, p^1_3\}$ and $\{p^1_3, p^1_4\}$ are present in this sandwich graph, resulting in an induced C_5.

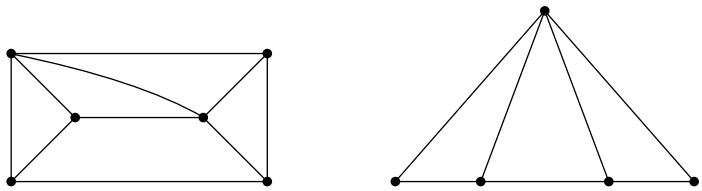

Fig. 4. On the left-hand side the graph P^c_6 and on the right-hand side the graph $\{P_4 + P_1\}^c$.

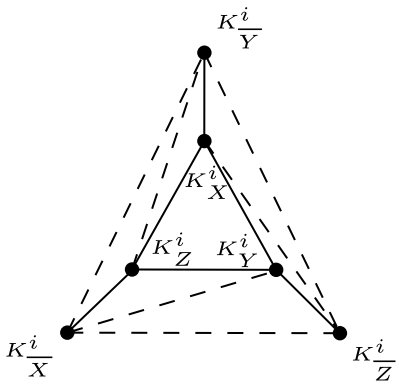

Fig. 5. A gadget which corresponds to the clause $(X + Y + Z)$ in the construction from [2]. Solid edges are forced, dashed edges are optional, and omitted edges are forbidden.

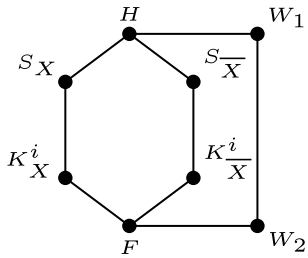

Fig. 6. Adding a four-vertex path in the construction from [2]. Both the vertices W_1 and W_2 have degree two in G'_2.

obtain a truth assignment for (U, C), and a truth assignment for (U, C) *indicates a set of optional edges* that can be added to the sandwich instance in order to triangulate each forced six-cycle corresponding to a variable.

Suppose that the sandwich instance described above has a chordal completion G. Then the truth assignment for (U, C) which we obtain by G, as described above, is satisfying [2, see Lemma 2].

Conversely, suppose that we have a satisfying truth assignment for (U, C). Then we add to the corresponding sandwich instance the set of optional edges which is indicated by this truth assignment in order to triangulate each forced six-cycle. To finish the chordal completion [2, see Lemma 2], the following sets of optional edges are added:

- true shoulders that is, shoulders S_X where X is true and shoulders $S_{\overline{X}}$ where $\overline{X}$ is false, form a clique;
- true knees that is, knees K^i_X where X is true and knees $K^i_{\overline{X}}$ where X is false, form a clique; and
- each true shoulder is adjacent to every knee.

Next we show how to modify the construction given in [2] which we described above in order to establish that the Even-Hole-Free-Sandwich-Problem is NP-complete. We add two auxiliary vertices W_1 and W_2 to the vertex set V (Fig. 6), obtaining $V' = V \cup \{W_1, W_2\}$. We add three forced edges to the forced edge set E_1, such that we have a four-vertex path induced by $\{H, W_1, W_2, F\}$, obtaining $E'_1 = E_1 \cup \{HW_1, W_1W_2, W_2F\}$. All other edges incident to vertices W_1, W_2 are forbidden edges in E'_3. In what follows we refer to the graphs (V', E'_1) and (V', E'_2) as G'_1 and G'_2, respectively.

We show that there is a satisfying truth assignment for (U, C) if and only if there is a sandwich graph G for (G'_1, G'_2), such that G is even-hole-free.

Suppose there is a satisfying truth assignment for (U, C). Consider the sandwich graph G obtained by adding the optional edges added in [2] as described above. Recall that the subgraph of G induced by $V = V' \setminus \{W_1, W_2\}$ is a chordal graph. Suppose G contains an even hole B. Then B contains both auxiliary vertices W_1 and W_2, and also vertices H and F. If the hole B contains a true shoulder s, then since s is adjacent in G to H and F, we have that $|B| = 5$. Otherwise, if the hole B contains a true knee k, then since k is adjacent to H and F, again we have that $|B| = 5$. Therefore, B contains a path from H to F using only the non-true shoulders and non-true knees. However, the set of non-true shoulders and non-true knees is an independent set, with shoulders only adjacent to H, and knees only adjacent to F, so no such path exists. We conclude that G is even-hole-free.

Suppose there is a sandwich graph G that is even-hole-free. The six-hole induced by $\{H, W_1, W_2, F, K^i_X, S_{\overline{X}}\}$ in G'_1 implies that the sandwich graph G contains at least one of the optional edges HK^i_X or $FS_{\overline{X}}$. If $HK^i_X \in E(G)$, then $HK^i_{\overline{X}} \notin E(G)$, as otherwise we have a C_4 induced by $\{H, K^i_X, F, K^i_{\overline{X}}\}$ in G. We claim that $FS_X \in E(G)$. Suppose not, then the set $\{H, W_1, W_2, F, K^i_{\overline{X}}, S_X\}$ induces a six-hole in G, which is a contradiction. Hence, $FS_X \in E(G)$. We conclude that if $HK^i_X \in E(G)$, then $FS_X \in E(G)$, which in turn implies $S_XK^i_X \in E(G)$, as otherwise we have a C_4 induced by $\{H, K^i_X, F, S_X\}$. We call the three added optional edges $HK^i_X, FS_X, S_XK^i_X$ the *positive* orientation. Otherwise $FS_{\overline{X}} \in E(G)$, and then $FS_X \notin E(G)$, as otherwise we have a C_4 induced by $\{H, F, S_X, S_{\overline{X}}\}$. Now the six-hole induced by $\{H, W_1, W_2, F, K^i_{\overline{X}}, S_X\}$ implies that $HK^i_{\overline{X}} \in E(G)$. We conclude that if $FS_{\overline{X}} \in E(G)$, then $HK^i_{\overline{X}} \in E(G)$, which in turn implies $S_{\overline{X}}K^i_{\overline{X}} \in E(G)$, as otherwise we have a C_4 induced by $\{H, F, S_{\overline{X}}, K^i_{\overline{X}}\}$. We call the three added optional edges $FS_{\overline{X}}, HK^i_{\overline{X}}, S_{\overline{X}}K^i_{\overline{X}}$ the *negative* orientation. All six-cycles corresponding to variable X must be oriented in the same way, which defines the truth assignment of X to be true or false.

To prove that we have defined a satisfying truth assignment, assume that there is a clause $i = (X + Y + Z)$ with three negative literals. The four-cycle $\{H, K^i_{\overline{Z}}, F, K^i_{\overline{X}}\}$ which is formed by the optional edges which are assumed to be present in G implies that the optional edge $K^i_{\overline{Z}}K^i_{\overline{X}}$ must be present in G as well (see Fig. 7). By symmetry, $K^i_{\overline{X}}K^i_{\overline{Y}} \in E(G)$. The four-cycle induced by $\{K^i_{\overline{Z}}, K^i_Y, K^i_Z, K^i_{\overline{X}}\}$ implies the optional edge $K^i_YK^i_{\overline{X}} \in E(G)$. We conclude that G contains a four-hole induced by $\{K^i_X, K^i_Y, K^i_{\overline{X}}, K^i_{\overline{Y}}\}$, which is a contradiction. □

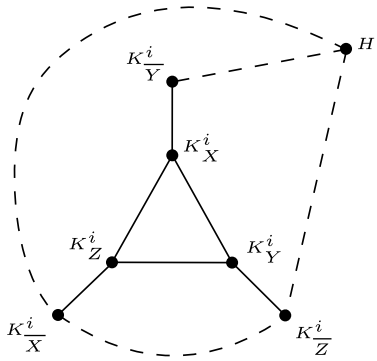

Fig. 7. If $i = (X + Y + Z)$ is a clause with three negative literals and G the corresponding sandwich graph, then the four-cycle $\{H, K^i_{\overline{Z}}, F, K^i_{\overline{X}}\}$ which is formed by the optional edges which are assumed to be present in G implies that the optional edge $K^i_{\overline{Z}}K^i_{\overline{X}}$ must be present in G as well. Solid edges are forced and dashed edges are the optional edges known so far to be present in G.

4. Concluding remarks

Almost monotone properties were introduced in [4], where the sandwich problem was proved to be in P in case the recognition problem was known to be in P. The properties of containing an odd hole and of containing an even hole were proved to be almost monotone. At that time, even-hole-free graphs and Berge graphs were known to be recognizable in polynomial time [3,7], and the polynomial-time recognition of odd-hole-free graphs was established later [6]. Together with our results, this implies that the two properties we consider, being odd-hole-free and being even-hole-free, are not almost monotone.

In the Not-$\mathcal{C}$-Free-Sandwich-Problem, we are asking if there exists a sandwich graph in which there exists an induced subgraph isomorphic to a graph in $\mathcal{C}$, whereas in the $\mathcal{C}$-Free-Sandwich-Problem, we are testing if there exists a sandwich graph G such that for every induced subgraph H of G, H is not in $\mathcal{C}$. The latter problem has an additional alternation, which is an indication that the Not-$\mathcal{C}$-Free-Sandwich-Problem might always be easier than the $\mathcal{C}$-Free-Sandwich-Problem. Clearly, if the $\mathcal{C}$-Free-Recognition-Problem is NP-complete (for example if $\mathcal{C}$ is the set of prisms), then the Not-$\mathcal{C}$-Free-Sandwich-Problem is NP-hard. This leads to two open questions:

- Is there a class of graphs $\mathcal{C}$ such that the $\mathcal{C}$-Free-Recognition-Problem is in P, but the Not-$\mathcal{C}$-Free-Sandwich-Problem is NP-complete?
- Is there a set $\mathcal{C}$ such that the $\mathcal{C}$-Free-Sandwich-Problem is in P, but the Not-$\mathcal{C}$-Free-Sandwich-Problem is NP-complete?

Truemper configurations [12], such as prisms, thetas, and pyramids, were considered for the Not-$\mathcal{C}$-Free-Sandwich-Problem. Both the Prism-Free-Sandwich-Problem and Not-Prism-Free-Sandwich-Problem are NP-complete, because the Prism-Free-Recognition-Problem is NP-complete [11]. However, the Theta-Free-Sandwich-Problem is NP-complete [8], but the Not-Theta-Free-Sandwich-Problem is in P [4]. The Not-Pyramid-Free-Sandwich-Problem is in P [4], but the complexity of the Pyramid-Free-Sandwich-Problem remains open. The complexity of the Perfect-Graph-Sandwich-Problem remains one of the most prominent open questions in this area [9].

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

We would like to thank the anonymous reviewers for their helpful comments which improved the presentation of this paper.

Data availability

No data was used for the research described in the article.

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